Constructing mathematical meanings with digital tools: design, implementation and analysis of a teaching activity in a distance education context

Eleonora Faggiano, Federica Mennuni

Dipartimento di Matematica, Università di Bari Aldo Moro, Via Orabona 4, 70125 Bari, Italy
eleonora.faggiano@uniba.it
federica.mennuni@uniba.it

Abstract. This paper presents and analyses a teaching experiment, carried out with five 12^{th} grade students, in a distance education context during the pandemic, in May 2020. The teaching activity, framed by the Theory of Semiotic Mediation, involved the use of a digital tool - a Dynamic Geometry Environment - and aimed at the construction of the mathematical meanings of rotation. Results are discussed in order to show whether and how the designed teaching activity, experimented in a distance education context, resulted to be effective in order to foster the students' construction of meanings. We are willing to contribute our findings to educational discussions in Covid-19 time, offering an example of integration of technology in mathematics teaching and learning, and focusing on theoretical aspects and methodological approaches that can be taken into consideration in the attempt to create meaningful technology-rich learning environments, both in remote teaching and in a traditional educational context.

Keywords: Construction of Mathematical Meanings, Distance Education, Theory of Semiotic Mediation, Dynamic Geometry Environments

1 Introduction

Within the context of the pandemic, the education system worldwide is facing the challenge of providing distance educational opportunities: at its most basic level, distance education can be said to take place when a teacher and student(s) are separated by physical distance, and the gap is bridged thanks to the use of technology (i.e., voice and/or video to communicate and share didactical materials, chats and social networks to facilitate collaboration and group working…).

In spite of the recent abrupt lead-in of technology in the educational processes, it is worthy of note that the transformations it induces affect ways of teaching and learning and requires new competences to be swiftly developed [1]. However, as underlined by Mously, Lambidin, and Koc [2], it cannot be taken for granted that technological advances alone can change essential aspects of teaching and learning, simply because they can bring about opportunities for change in pedagogical practice. Furthermore, teachers are usually concerned with [3] and trained [4] to use specific technological tools, and during the pandemic the focus seems to be mainly on those tools which allow the basic level of distance education to take place. However, in accordance with Harasim [5], we believe that restricting the use of digital technologies to make traditional didactic teaching easier or more efficient means missing opportunities to introduce better, different or more advanced ways of learning. A theory-informed approach, that rarely seems to characterize teacher professional development programs, could instead contribute to transform educational practices [6].

The pandemic, hence, can be considered as an occasion to rethink and reassess teaching practices and pedagogical approaches in relation to the opportunities afforded by digital technologies. In this sense, as far it concerns the remote educational context, this paper would offer a contribution to research, focusing on the particular case of mathematics.

In the case of technology as a crucial tool in supporting the educational processes in mathematics, a substantial amount of research has proved that the use of technology may allow teachers to create suitable learning environments, with the goal of promoting the construction of meanings for mathematical objects [7], [8], [9], [10].

However, as highlighted for example by the Italian Committee for Mathematics Education [11]:

> The meaning cannot be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself. (p. 32)

The construction of mathematical meanings is an aspect of fundamental importance in the research field of mathematics education [12], [13] and effective distance education cannot fail to take this into account.

Moreover, in order to foster the construction of mathematical meanings, teaching and learning need to be radically transformed so that technological tools can be effectively integrated into educational activities. This requires teachers to become aware of the potential usefulness and effectiveness of technological tools as a pedagogical resource [3], [14]. The role of the teacher in fostering the construction of mathematical meanings, indeed, is fundamental in order to guide the evolution of personal meanings, which emerge during the accomplishment of mathematical tasks, towards shared mathematical meanings.

This paper attempts to contribute to the reflection on remote teaching due to the pandemic, showing that, even in a remote context, the construction of meanings can be promoted by didactic activities that exploit the potential of technologies. To do that, we present and analyze a teaching activity, experimented in a distance education context during the pandemic. In particular, we aim to stress some theoretical aspects and methodological approaches that can be taken into consideration with the attempt to create meaningful technology-rich learning environments, both in remote teaching and in a traditional educational context.

2 The construction of mathematical meanings within technology-rich learning environments

This study is based on the idea of a mathematics laboratory, which comes from both empirical and theoretical studies in face-to-face classroom context but is perfectly in tune with the context of distance education. In the UMI-CIIM document [11], indeed, it is summarized as follows:

> A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to [promote] the construction of meanings of mathematical objects. (p. 32)

In this sense, a laboratory environment can be seen as a Renaissance workshop, in which the apprentices learn by doing and communicating with each other about their practices. In particular, in the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools; and on the other, to the interactions between people working together, without distinguishing between the teacher and the students.

This idea of a mathematics laboratory, therefore, calls for the creation of suitable and meaningful learning environments, in which technological tools assume a crucial role in supporting the teaching and learning processes. However, as Laborde [15] underlined already in 2002, the introduction of technology in complex teaching systems produces perturbation and, hence, for a teacher to ensure a new equilibrium, he/she needs to: understand why and how learning might occur in a technology-rich situation and become able to create appropriate learning situations, making adequate, non-trivial choices.

The topic of the construction of mathematical meanings with digital tools, in particular, has been extensively studied, at least for the last twenty years [16], [17], [18]. The issue has been addressed from different prospective such as: the design and development of the resources [19]; the mathematics curriculum development and task design [20]; the benefit for the students' learning [21]; and the mathematics teacher's education and professional development [8]. The emergency of the pandemic can be considered as an interesting occasion to bridge the gap between research and practice concerning the integration of technology in the teaching and learning processes. In particular, we willingly contribute the findings of this paper to this issue.

For this reason, in the next two subsections we will focus respectively on some theoretical aspects and methodological approaches on which the teaching activity, we present and discuss in this paper, is based.

2.1 The Theory of Semiotic Mediation and the role of the teacher

Many studies in the field of mathematics education have proved that an effective research tool to design, implement and analyze teaching activities aiming at fostering the construction of mathematical meanings through the use of tools can be found in the Theory of Semiotic Mediation (TSM) [22].

Elaborated by Bartolini Bussi and Mariotti [22] from a Vygotskian point of view, this theoretical framework also allows us to exploit the affordances of a suitable integration of technology in the learning environment, focusing on the mediating role of the tool. The aspects of the TSM which brought us to choose this framework among others (such as the Technological, Pedagogical and Content Knowledge (TPACK) [23] the Instrumental Orchestration [24] or the Structuring Features of Classroom Practice [25]) is that it elaborates the complex notion of mediation combining a semiotic and an educational perspective. It takes into account the epistemological issue concerning the relationship between the accomplishment of a task and the student's mathematical learning processes and considers the crucial role of human mediation [26] in the teaching-learning process [27].

The TSM, indeed, considers the complex system of semiotic relations between the elements involved in the construction of mathematical meanings through the use of artefacts. An artefact is seen as a device in itself, and it is counterposed to an instrument which comprises also the involved schemes to use it [28].

Although the notion of artefact is central in the TSM, for the purpose of this paper we will refer to those that, in terms of TSM, we should call the digital and non digital artefacts, using for them the more common word "tools".

The two key elements of the TSM are the notion of semiotic potential of an artefact and the notion of didactic cycle. The semiotic potential of an artefact is the twofold relationship that the artefact has with the personal meanings emerging from its use, and the mathematical meanings that might be evoked by such use. The term didactic cycle refers to the organization of teaching in three different steps of activities: activities with the artefact, activities of individual production of signs, and finally collective discussions [29] (see Fig. 1).

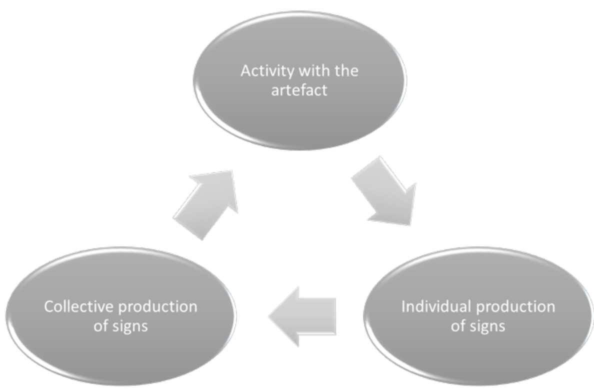


Fig. 1.. The didactic cycle in the Theory of Semiotic Mediation.

According to the TSM, personal meanings emerging from the activities carried out with an artefact may evolve into mathematical meanings, in peer interaction during the accomplishment of the task and in the collective discussions, orchestrated by the teacher. A sequence of didactic cycles can be organized with the aim of supporting this evolution.

In order to foster the shared construction of meanings, hence, in each of the didactic cycle of a teaching activity, students are asked to: carry out a task involving the use of the artefact; describe what they made accomplishing the task with the artefact, explaining what happened and giving interpretations; discuss the carried-out experience in order to collectively construct the mathematical meanings, with the mediation of the teacher. This complex process, in which the teacher uses the artefact as a tool of semiotic mediation, allows signs to emerge and finally evolve towards mathematical meanings. For this reason, the role of the teacher is fundamental in the learning process and, hence, also in our teaching experiment.

Although the Theory of Semiotic Mediation has been extensively studied within the mathematics education research field, less is known about its implementation in remote teaching experiences. This paper attempts to address this gap, presenting and analyzing a teaching experiment, framed by the TSM, developed in a distance educational context, in which the digital tools are used as semiotic mediators and the teacher takes the fundamental role to foster the collective construction of mathematical meanings.

2.2 Dynamic Geometry Environments and the role of digital tools

With the attempt to investigate the use of digital technology to construct mathematical meanings, we focus on a particular kind of tool which is a research topic in mathematics education since the eighties: Dynamic Geometry Environments (DGEs). A DGE is a computational microworld, embedding Euclidean Geometry, in which it is possible to construct geometric figures and interact with them. The main potential of the DGEs is based on the possibility to drag geometric objects and observe the effects of the dragging: the independent elements of the construction can be dragged in order to observe if relationships remain intact (confirmatory dragging) or whether any properties of the figure remain invariant (exploratory dragging) [30], [31].

This typical characteristic, usually called the “dragging function”, allows experimenting with dynamic and interactive modes of visualization and exploration [32], and can be instrumental in helping students to solve construction problems and to formulate conjectures [33].

Findings from many research studies (see, for instance [34]) suggest that the characteristics of the DGEs (e.g., measuring, dragging, and commands for constructing) allow students to conjecture and generalize mathematical relationship and to be engaged in key mathematical processes. For these reasons DGEs can be used as semiotic mediators in the learning processes [22], [35].

Despite the large number of studies concerning the use of DGEs in mathematics education, however, their use in the distance education context has been far less investigated. With respect to this gap, with this paper we do not intend to compare an online teaching approach using a DGE with a face-to-face teaching, but rather we want to offer an example of effective integration of DGEs in mathematics remote teaching and learning. In our view, indeed, the contribution of this paper lies in its aim to stress that the potentiality of this kind of tools to foster the construction of mathematical meanings, could be discovered during the emergency remote teaching and then, hopefully, applied also in the traditional educational context.

3 Method

The methodology used for the research presented in this paper is that of the Teaching Experiment suggested by Steffe and Thompson [36], according to which theoretical hypotheses guide the design of a teaching intervention, whose implementation and evaluation allows us to validate or refine (possibly reject) these hypotheses. The design of the teaching sequence of tasks, in accordance with the chosen theoretical framework and, with the formulated hypotheses, plays a key role in this methodology and it is associated with the analysis of all the data generated during the teaching experiment. In the specific case of this research, the activity with digital and non digital tools, in accordance with the TSM, must lead to the individual production of signs. These signs must then be shared in discussion activities orchestrated by the teacher, in order to reach the collective production of signs and their evolution towards mathematical signs. To do this, a teaching sequence on rotations was designed for high school students and was implemented in a first pilot study, involving a small group of students during the pandemic. The analysis of the results of the experiment will constitute the basis for a further study to be conducted with a whole class, highlighting, in particular the role of e-learning platform features, with respect to the mathematical and pedagogical aims.

3.1 Research question

In previous research [37] we have already shown how digital tools can act as mediators between the experience and the mathematical conceptualization. In this paper we investigate whether and how the use of digital tools, eventually combined with other non digital tools, can contribute to foster the construction of mathematical meanings, also in the case of distance education context.

In particular, we attempt to answer the following research question:
Can a teaching activity framed by the TSM and implemented in a context of distance education, foster the construction of mathematical meanings?

3.2 Participants, procedure and data collection

The teaching activity was experimented, during the Covid-19 lockdown, with the participation of five 17-years old students, attending the 12th grade in Italy. The teaching experiment was developed in two distance lessons of three hours each, within a University program aimed at engaging high school students in innovative learning experiences.

In both the lessons students were asked to accomplish a sequence of three tasks on rotation, involving different kind of tools. According to the TSM, at the end of each of the activities a collective discussion was conducted by the teacher, aimed at allowing personal meanings to emerge and evolve towards the meaning of rotation.

The lessons, carried out through the Zoom platform, were both recorded, transcribed and analyzed, together with the students' protocols.

Overview of the teaching sequence. The design of the sequence was framed by the TSM, taking into account the semiotic potential of the artefacts with respect to the aims of the teaching intervention and the formulations of the tasks [38]. In this section, we present an overview of the teaching sequence, consisting of six tasks, aimed at developing the mathematical meaning of rotation as an isometric transformation, characterized by specific properties. In particular, after the introduction of the first two tasks, we will focus on the third and the fourth tasks, without presenting the details of the last two tasks, whose aims are more connected with the mathematical content and move the discussion beyond the purpose of this paper.

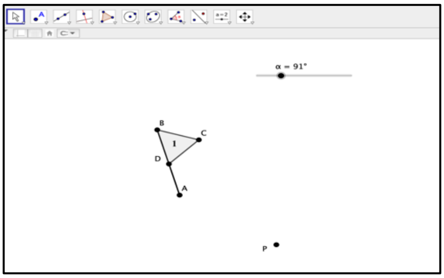
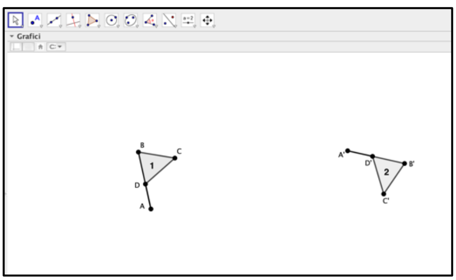
The aim of the first two tasks was to introduce the mathematical meaning of rotation as a one-to-one correspondence of points in the plane, highlighting its dependence on the center and on the angle, and the distance preservation of each pair of corresponding points from the center. In the first task, students were required to overlap a transparency on a printed paper containing a figure and two points and to let it rotate 90 degrees around a pin, fixed in one of the two points. The figure was reproduced on the transparency in each of its obtained position so that the transparency finally contained the original figure and the two rotated figures with respect to the two points. At the end of this construction, students were asked to answer appropriate questions in order to explain the initial properties of the rotation. The request was, in particular, to compare the two rotated figures with each other and with the initial figure. In the second task students were required to draw the figure obtained by rotating 90 degrees clockwise a given figure around a given (external) center, and to explain the way they did it. In order to carry out the request, the students could use tools such as the protractor, a ruler, set squares and a compass.

The aims of the third task, which required the use of a DGE (GeoGebra – www.geogebra.org), were to draw students' attention on the meaning of rotation as an isometric transformation and to highlight, in particular, the fundamental roles played by the center and the angle of rotation.

The aim of the fourth task, which again required the use of GeoGebra, was to draw students' attention to the center of the rotation as the unique point at the same distance from each pair of corresponding points of the figures. That is, the center of rotation can be found as the intersection point of the perpendicular bisectors of any two segments joining a point of one figure to the corresponding point of the other figure.

Table 1 presents the semiotic potential of the tools in correspondence to the third and the fourth task. In particular these two tasks are described presenting the given tools, a screenshot of the GeoGebra file used to accomplish the task and the requests given to the students.

Table. 1.. The third and the fourth task with the related semiotic potential of the tools.

Task	**Description of the tasks**	**Semiotic potentials of the tools**
III	Students were given a figure, an external point P and a slider, that can be used to vary the angle α of rotation in a fixed range.  They were asked to: - use the tool/button "Rotate around point" in order to construct the rotated figure obtained by rotating the given figure around P, selecting the angle on the slider; - observe what moves and what doesn't move when changing the angle using the slider, or when dragging the given figure or the point P; - explain the reasons of the behaviors.	- clicking on the button/tool 'Rotate around point' and then choose a point of the figure, the center P and the angle α on the slider, evokes the idea of rotation as a correspondence of points; - observing that the distances between the points of the rotated figure with respect to the center vary when dragging the given figure or the point P, evokes the idea of the dependence of the rotated figure on the given figure and on the center and of the distance preservation of each pair of corresponding points from the center; - observing the movement of the rotated figure alongside a circumference when changing the angle on the slider, evokes the idea of rotation as a circular rigid movement of a figure around the center by a certain angle.
IV	Students were given two congruent figures.  They were asked to: - find the center of the rotation that transforms one figure into the other; - explain how the point has been identified.	- observing that the perpendicular bisectors of any segments joining pairs of corresponding points intersect at a unique point, evokes the idea that the center of the rotation is the unique point at the same distance from each pair of corresponding points of the figures.

The last two tasks aimed at investigating more deeply some properties of the rotations. It was done involving students in conjecturing, verifying and proving.

3.2 Data analysis

With respect to the aim of this paper, we will report a series of interactions among the students and the teacher, referring to the digital tool and focusing in particular on the third and the fourth tasks, in order to describe and explain a series of conceptual steps.

With the analysis of the results we will try to figure out how the interactions throughout the activities with the use of the digital tool, together with the guidance of the teacher during the collective discussions, were involved in the construction of the mathematical meanings. The analysis will show that, also in a context of distance education, a teaching activity framed by the TSM and involving the use of digital and non digital tools, could foster the construction of mathematical meanings thanks to the fundamental role of the teacher.

4 Results

The first episode we present is extracted by the discussion after the students' accomplishment of the third task. They had used the GeoGebra button to obtain the rotated figure and the teacher asked them what they have observed when varying the angle α using the slider and activating the trace on the points A and A' (Fig. 2):

I-02:05:47 V: When the angle α changes, we see that A remains fixed in its position, while A' changes its position and its trace draws an arc of circumference. And then we have the center of rotation P, that remains fixed, and the distances of A and A' from P, that are constant. Hence, these distances have to be equal.

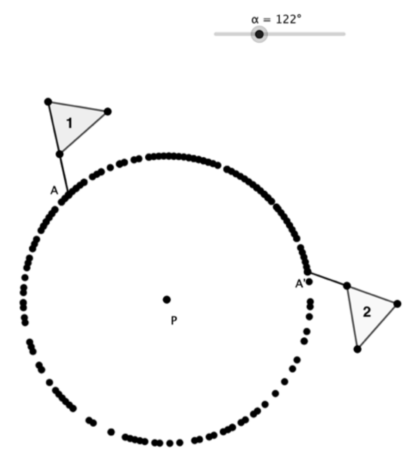


Fig. 2.. Result of the trace drawn by the movement of point A' when changing the angle using the slider.

The next question required the students to drag the given figure (labelled with "1") and to observe what moves and what doesn't move (Fig. 3).

I-02:07:28 I: I saw that if, for example, we try to bring flag 1 closer to, or even move away from the center of rotation P, we will see that flag 2 will also move in the same way. The new distance between A and P will be equal to the new distance between A' and P' and then... we see that the angle α does not change, it is always the same... and then P remains unchanged, it does not move.

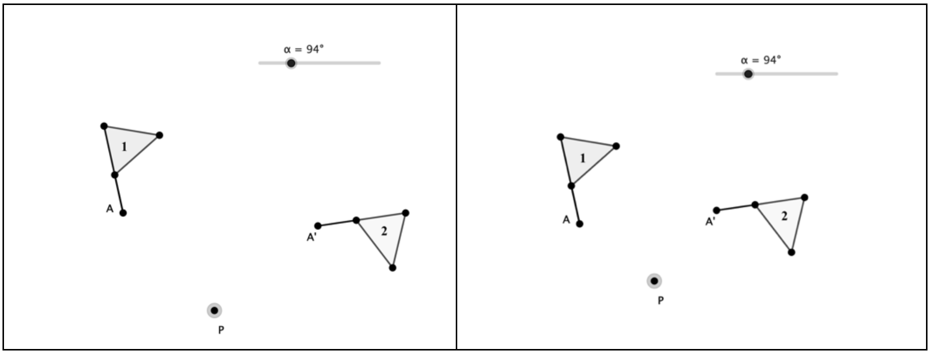


Fig. 3.. Two successive screenshots, taken during I.'s explanation of the effect of the dragging of the given figure (1).

In the final question students were required to drag the center of rotation P and, again, to observe what moves and what doesn't move.

I-02:09:20 P: Moving the point P, we see that the rotated flag (2) is displaced from its initial position, but the angle always remains the one we initially chose. So, the angle is the same. What changes is the position of flag 2, because we have dragged the center of rotation P.

The discussion ended with the teacher intervention aimed at summing up the definition of rotation and the relation among the objects:

I-02:11:32 Teacher: And so, how do we define a rotation?

I-02:11:47 V: A rotation is a geometric transformation such that, given the center P and the angle α of rotation, it associates to each point A of the plane a point A'. It is therefore the geometric transformation that associates to point A a point A', such that PA is congruent with PA' and P is the center of rotation, and the angle APA' is congruent to α.

I-02:18:58 Teacher: So, in your definition of rotation, dependencies are implicit, right? what does the rotated figure depend on?

I-02:19:18 E: On the center and the angle of rotation, and on the position of the given figure.

Below we present some excerpts of the discussion after the fourth task. None of the students correctly identified the hidden center of the rotation which transformed one figure into the other. V. firstly pointed on a point in the GeoGebra plane that seemed to be the center and then verified if the properties were still satisfied: she draw the circumferences with center in the hypothetic center of rotation, passing through the main points of the first figure, to verify if the corresponding points of the second figure belong to the relative circumferences; then she checked if the obtained angles of rotation were all equals (Fig. 4). It was from the last test that the doubts arose, and the discussion started.

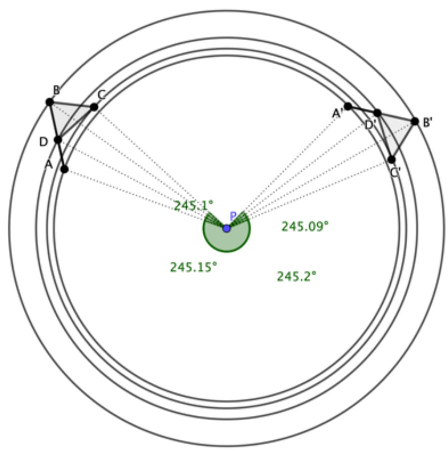


Fig. 4.. V's test to verify if the identified center of rotation is the correct one.

During the discussion one of the students, P., explained that he considered the midpoint between A and A', even if he then realized that it couldn't be the center of rotation.

The teacher intervention brought V. to think at P's words and so to consider the perpendicular bisectors of the segments joining corresponding pairs of points (Fig. 5):

II-00:12:18 Teacher: Try to think for a while... V. was saying something about distances… Last time you started to see some of the properties of rotation, right?... So, try to focus on the properties for a moment and, if you like, read them backwards, because it is the properties that can help you find the center.

II-00:13:53 V: We could draw the segments AA', BB', CC' and DD'… and consider the perpendicular bisectors of each of them… Then the center should be… this point [she pointed on the common intersection points of the four perpendicular bisectors]

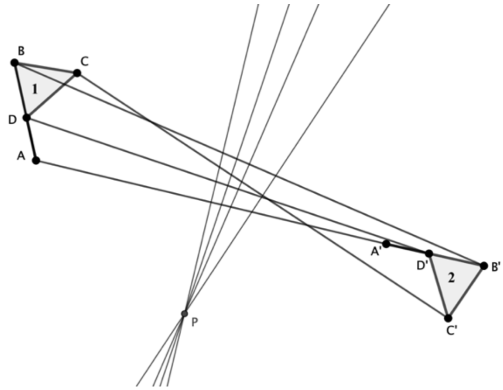


Fig. 5.. V's attempt to identify the center through the perpendicular bisectors of the segments joining pairs of corresponding points.

The teacher asked her to explain her reasoning:

II-00:16:26 Teacher: Why did you choose these four perpendicular bisectors?

II-00:16:33 V: Since P. had considered the midpoint, and since we know that one of the properties of rotation is that of the preservation of the distances of the points of the two figures from the center, I thought that ... the geometric locus of the points equidistant from the extremes of the segment AA', as well as for the other segments, it is the perpendicular bisector. So, if we intersect all the perpendicular bisectors, the point we get could be the center of this rotation

V's argumentation suggested that the group perform a validation asking GeoGebra to measure the amplitude of the angles APA', BPB' and so on, and the distances of the corresponding points from the center, in order to verify their equality (Fig. 6).

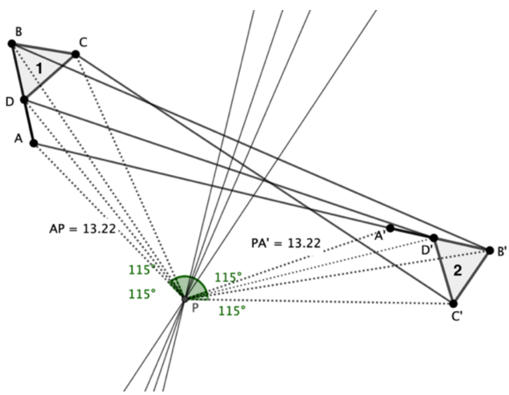


Fig. 6.. V's final test to verify if the identified center of rotation is the correct one.

The test carried out by V., and shared with the whole group using the sharing function in Zoom, succeeded in persuading the students that the center of rotation can be found as the intersection point of the perpendicular bisectors of any two segments joining a point of one figure to the corresponding point of the other figure. This drew their attention to the center of the rotation as the unique point at the same distance from each pair of corresponding points of the figures.

5 Discussion

The aim of this section is to show how the presented results allow us to answer our research question. In particular, we will show that: the teaching activity, framed by the TSM, resulted to be effective for personal meanings to emerge and evolve towards a shared and more comprehensive construction of the mathematical meaning of the rotation and its properties; the guidance of the teacher in conducting the discussion resulted to be important in order to give meanings to the properties of rotation; the digital tools, the DGE GeoGebra, resulted to be fundamental in fostering students to endow rotation with its mathematical meaning.

5.1 Effectiveness of the teaching activity and role of the teacher for the construction of the mathematical meaning

The episode extracted by the discussion on the third task, shows that the students didn't have difficulties in making a hypothesis and justifying the behavior of the geometrical objects according to the definition of rotation around a point (see Tab. 2). Indeed, they recognized that the variation of the angle, of the position of the given figure and of the center, all affect the position of the rotated figure, but given that in each of the cases, the distances from the center of each pairs of corresponding points remain equal (see: I-02:05:47, I-02:07:28, and I-02:09:20). This allowed them to recognize, thanks to the teacher's intervention, that a rotation around a point is an isometric transformation (see: I-02:11:32 and I-02:11:47) and to recognize, in particular, the fundamental role played by the center of rotation (see: I-02:18:58 and I-02:19:18).

Table. 2.. The episode extracted by the discussion on the third task with the related semiotic potential of the tool and the corresponding mathematical meanings constructed by the students

Task	Episode	Semiotic potential	Construction of the meaning
III	I-02:05:47	- clicking on the button/tool 'Rotate around point' and then choose a point of the figure, the center P and the angle α on the slider, evokes the idea of rotation as a correspondence of points; - observing the movement of the rotated figure alongside a circumference when changing	Rotation is a one-to-one correspondence of points and can be seen as a circular rigid movement of a figure around the center, by a certain angle,

		the angle on the slider, evokes the idea of rotation as a circular rigid movement of a figure around the center by a certain angle.	
III	I-02:07:28 I-02:09:20	- observing that the distances between the points of the rotated figure with respect to the center vary when dragging the given figure or the point P, evokes the idea of the dependence of the rotated figure on the given figure and on the center and of the distance preservation of each pair of corresponding points from the center;	The rotated figure depends on the given figure. The rotated figure depends on the center and the angle of rotation. The distance of each pair of corresponding points from the center is preserved.
III	I-02:11:47 I-02:19:18	- clicking on the button/tool 'Rotate around point' and then choose a point of the figure, the center P and the angle α on the slider, evokes the idea of rotation as a correspondence of points;	Rotation is a geometric transformation in which a fundamental role is played by the center and the angle of rotation.

Being aware of the potential of the digital tool, both in terms of mathematical meanings and in terms of the students' production of personal meanings, the teacher's aim was to lead, with appropriate questions, the evolution of personal meanings towards the mathematical meanings.

In the presented episodes, the teacher's intervention was fundamental in inducing students to make explicit the personal meanings, created while accomplishing the tasks. On this basis, with her intervention she addressed the shared reformulation of students' meanings in order to foster their evolution towards more complete mathematical meanings concerning the concept of rotation.

During the discussion of the third task, the teacher's intervention to summarize, with the request to make explicit the definition of rotation (see: I-02:11:32) and the implicit dependencies between the geometric objects (see: I-02:18:58), proved to be useful for students to focus on the meaning of rotation and its properties, and in particular on the fundamental role played by the center, that would have been important to tackle with the next task.

During the discussion of the fourth task, the teacher's intervention resulted to be important in determining the center of rotation starting from the mathematical meanings of rotation, which had come out with the previous activities. With her interventions during the discussion, the teacher tried to bring to the fore all the students' personal meanings in order to lead their evolution (see: II-00:12:18).

In fact, with her intervention, the teacher pushed the students to connect the idea of one of them, considering the midpoint of the segments obtained by joining pairs of corresponding points, with the property of the rotation explained by V. on the preservation of the distances from the center (see Tab. 3). Therefore, this resulted crucial in order to guide the students to accomplish the task and brought the same student V. to realize that she must use the perpendicular bisectors (see: II-00:13:53).

Finally, the teacher's request to let student V. share her reasoning (see: II-00:16:26) was also fundamental for all the other students: in this phase of the collective discussion it allowed to generalize the property of equidistance, characterizing the rotation, starting from the idea of having to use the midpoint, and arriving at the use of perpendicular bisectors, and to determine the center of rotation as the intersection point of any two of them (see: II-00:16:33).

Table. 3.. The episode extracted by the discussion on the fourth task with the related semiotic potential of the tool and the corresponding mathematical meanings constructed by the students

Task	**Episode**	**Semiotic potential**	**Construction of the meaning**
IV	II-00:13:53 II-00:16:33	- observing that the perpendicular bisectors of any segments joining pairs of corresponding points intersect at a unique point, evokes the idea that the center of the rotation is the unique point at the same distance from each pair of corresponding points of the figures.	The center of rotation can be found as the intersection point of the perpendicular bisectors of any two segments joining a point of one figure to the corresponding point of the other figure

5.2 The role of the digital tool for the construction of the mathematical meaning

Concerning the role of the digital tool, the discussion on the third task during the first lesson, demonstrated its effectiveness in order to let the students' personal meanings emerge. The reason can be found in the characteristic of the DGE to embed the Euclidean Geometry and, in particular, in the meaning of rotation embedded in the "Rotation around a point" button that the students used in the construction. The constraints imposed by the mathematical meaning of rotation, indeed, are those that allow the construction of the rotated figure to resist to the dragging and to reveal the relationship among the objects and the properties of the rotation. However, it is especially in the episode extracted by the discussion on the next task that the role of the digital tool in fostering the evolution of meanings becomes evident. The opportunity given by the DGE to exploit the measurement functions allows students to face up to the error and so to reconsider the task from another point of view (see Fig. 4): the identification of the center became a matter of application of the same properties they recognized in the previous task. The equidistance of the corresponding points from the center was connected with the notion of perpendicular bisector, and preservation of the angle of rotation was connected with the need to consider at least two pairs of corresponding points. These two considerations allowed the meaning of the center of rotation to evolve towards the unique point at the same distance from each pair of corresponding points of the figures (see Fig. 6).

6 Conclusions

The episodes discussed in the previous section, show how the use of digital tool and the collective discussion guided by the teacher fostered the evolution of meanings: the students moved from what they have done to reflect on and to apply the properties of rotation, reaching a comprehensive knowledge of the content.

In fact, for personal meanings to emerge and evolve towards mathematical meanings, it was fundamental, as we have seen, the interaction of the students with the digital tool during the accomplishment of the given tasks. However, our findings point also to the effectiveness of the moments in which students have used Zoom's sharing function: it was then that they became more familiar with the concept of rotation and its properties, by comparing their work with each other and being guided by the teacher. The most evident aspect is the one related to the characterization of the center of rotation - as the intersection point of the perpendicular bisectors of any two segments obtained by joining pairs of corresponding points.

Therefore, we can say that the activity, framed by the Theory of Semiotic Mediation within a technology-rich environment, contributed to the construction of the mathematical meaning of rotation with its related properties, even if it was developed in a distance education context, thanks to the guidance of the teacher and the sharing function of the Zoom platform.

Results attempt to contribute to the emergent discussion unfolded by the pandemic, on the pedagogical and content knowledge required by teachers in order to tackle with the integration of digital technology in their practice, be they in presence or at a distance. We claim indeed, that from the theoretical point of view, teachers should be aware of the role of digital and non digital tools as semiotic mediators between experience and conceptualization. Moreover, from the methodological point of view, in order to foster the construction of meanings teachers need to become aware of the effectiveness of some practices, such as: those concerning the integration of digital and non digital tools in meaningful teaching and learning activities, within the context of mathematics laboratory; or the way to recognize the signs emerging through the use of tools in the accomplishment of the tasks, and to foster their evolution towards mathematical signs, within the context of a collective mathematical discussion.

The study presented and analyzed in this paper was developed with a small group of students. Results of a larger experiment, carried out with an entire class, are being analyzed.

A more in-depth study could also highlight the potential and limitations of different technical solutions for sharing students' productions during the group works and the collective discussions.

This study calls for necessary subsequent studies that deal with how the embodied dimension can be recovered at a distance. The embodied dimension, indeed, seems to do not contribute to the learning processes as it proved to happen in classroom context [37].

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